Lecture: Sudro Hall 27, TR: 8:00 - 9:15 AM

GENEVA MECHANISM

Indexing mechanisms are intermittent motion mechanisms that hold position alternately with a timed, unidirectional motion of the output member. This is distinct from other types of intermittent motion mechanisms such as dwell cams, which alternate forward and return motion with holding position. The output member of an indexing mechanism always advances in the same direction. Indexing mechanisms are practically important in such applications as weaving looms, in advancing workpieces in repetitive manufacturing operations, and in many instrument mechanisms.

The most common type of indexing mechanism is a Geneva mechanism. Geneva mechanisms come in many varieties, both planar and spherical. When advancing, it is kinematically similar to an inverted slider crank. When holding position, it functions as a simple journal bearing.

The name Geneva mechanism originated because these mechanism were used in mechanical watch and clock movements in the days when mechanical movements were dominant, and Switzerland was the world center of the industry.

A simple example of a Geneva mechanism is shown in Fig. 5.28. The pin, *P*, on the driving wheel engages the slots in the star-shaped driven wheel to advance the driven wheel one-quarter turn for every rotation of the driving wheel. In between the advance movements, the eccentric cylindrical journal surfaces cut into the star wheel engage with the journal surface on the driving wheel to lock the star wheel in position, although the driving wheel continues to rotate. The centerline of the slot must be tangent to the circle, with radius r, described by the center of the pin at the position in which the pin enters or leaves the slot. If this condition is not satisfied, there will be infinite acceleration at the beginning of advancement and infinite deceleration at the end. This condition dictates that the center distance of the two wheels should be $\sqrt{2}r$. It also requires that the outer radius of the star wheel be r. The radius of the journal surfaces is flexible. The centers of the cylindrical cutouts on the star wheel lie on a circle with radius $\sqrt{2}r$.

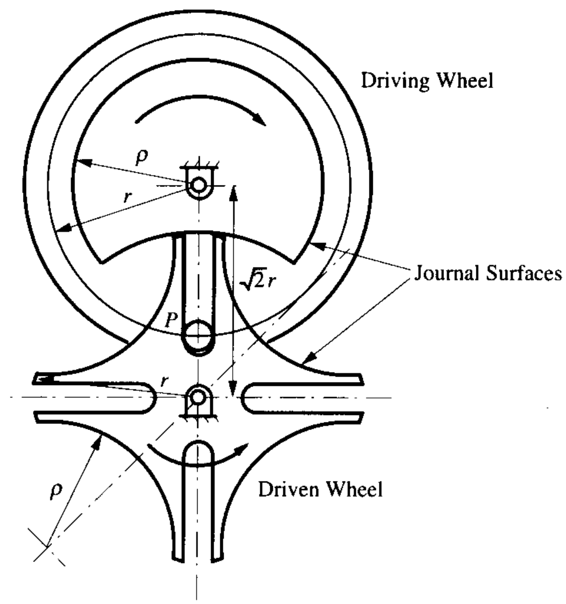

Figure 5.28 A four-station Geneva mechanism. The output member is the star wheel. The star wheel is advanced by the pin in the input wheel. The star wheel is advanced one-quarter of a revolution counterclockwise for every revolution of the input wheel. The advance movement occurs during one-quarter of a cycle with the star wheel being locked by the journal surface on the input wheel for the other three-quarters of the cycle.

During the advancing phase of the cycle, the mechanism is kinematically equivalent to an inverted slider crank. One of its attractions is that it smoothly accelerates and then decelerates the star wheel.

The motion of the star wheel may be analyzed by reference to Fig. 5.29. Resolving the sides of the triangle whose vertices are the two shaft axes and the pin axis in the vertical and horizontal directions:

$$\begin{aligned} r \sin \theta &= x \sin \phi \\ r \cos \theta + x \cos \phi &= \sqrt{2} r \end{aligned} \tag{5.25}$$

Elimination of x by substitution from the first of these equations into the second gives

$$\cos \theta + \frac{\sin \theta}{\tan \phi} = \sqrt{2}$$

after canceling the common factor, r. Rearrangement of this expression gives

$$\tan \phi = \frac{\sin \theta}{\sqrt{2} - \cos \theta} \tag{5.26}$$

or

$$\phi = \tan^{-1}\left(\frac{\sin\theta}{\sqrt{2} - \cos\theta}\right) \tag{5.27}$$

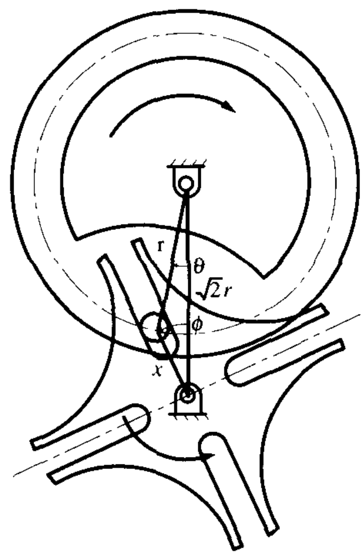

Figure 5.29 Kinematic modeling of the Geneva mechanism of Fig. 5.28. θ is the angle of rotation of the driving wheel, measured from the line of centers; ϕ is the angle of rotation of the star wheel.

Differentiation of Eq. (5.26) with respect to time followed by simplification gives

$$\dot{\phi}(1 + \tan^2\phi) = \dot{\theta}\frac{(\sqrt{2}\cos\theta - 1)}{(\sqrt{2} - \cos\theta)^2}$$

Substitution for $\tan\phi$ from Eq. (5.26) gives, after rearrangement and simplification,

$$\dot{\phi} = \dot{\theta}\left(\frac{\sqrt{2}\cos\theta - 1}{3 - 2\sqrt{2}\cos\theta}\right) \tag{5.28}$$

Differentiation again with respect to time gives, after simplification,

$$\ddot{\phi} = \ddot{\theta}\left(\frac{\sqrt{2}\cos\theta - 1}{3 - 2\sqrt{2}\cos\theta}\right) - \dot{\theta}^2\frac{\sqrt{2}\sin\theta}{(3 - 2\sqrt{2}\cos\theta)^2}$$

In the usual case in which the driving wheel is driven at constant angular velocity, the first term disappears and

$$\ddot{\phi} = \dot{\theta}^2\frac{\sqrt{2}\sin\theta}{(3 - 2\sqrt{2}\cos\theta)^2} \tag{5.29}$$

Equations (5.27), (5.28), and (5.29) are plotted versus θ (in degrees) in Fig. 5.30; ϕ is plotted in radians. Of course, ϕ varies from $-45°$ to $45°$ during the advancement. The angular velocity curve is actually $\dot{\phi}/\dot{\theta}$, and the angular acceleration curve is $\ddot{\phi}/\dot{\theta}^2$.

As can be seen from Fig. 5.30, the velocity and acceleration curves are smooth and well behaved, but the derivative of the acceleration (jerk) is infinite at the beginning and end of the advancement. So far, we have considered only the simplest version of the Geneva mechanism: the four-station planar variety. The number of stations is the number of slots in the star wheel and may, in principle, be any number, although the geometric lower limit is three. There is also a practical upper limit at which the journal surfaces on the star wheel become too short to effectively lock the output between advancements. The number of pins on the driving wheel is usually one, but drivers with two or more are possible.

The essential geometry for relating the number of stations to the duration of the advancement is shown in Fig. 5.31. Here α is the angle between the slot centerline and the line of centers of the two wheels at the moment of engagement or disengagement of the pin.

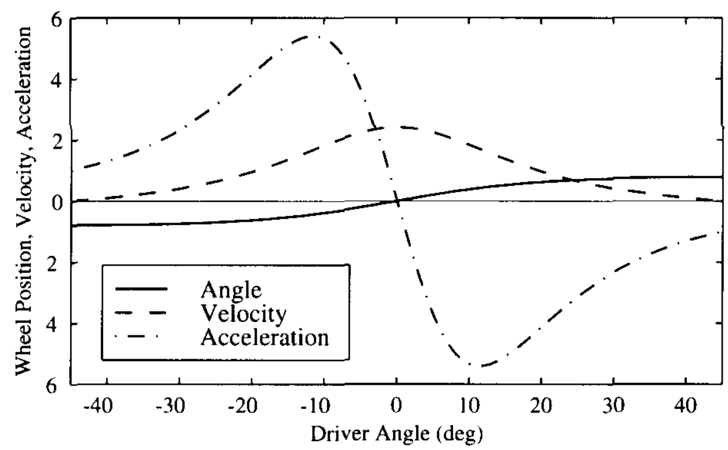

Figure 5.30 Position, velocity, and acceleration of the driven wheel of the Geneva mechanism shown in Figs. 5.28 and 5.29 during the advancement phase of the motion cycle. The angular position of the star wheel is in radians. The angular velocity and acceleration curves are respectively normalized to the driver angular velocity and driver angular velocity squared.

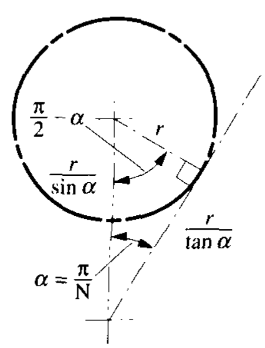

Figure 5.31 Critical geometry for a Geneva mechanism with N stations. α is the angle between the slot centerline and the line of centers at the moment of engagement of the pin; α is half the angle between successive slots on the star wheel.

That is, α is half the angle between successive slots, or $360°/(2N)$, where N is the number of stations. As already noted, the slot axis must be tangent to the circle traversed by the pin center at these positions in order to avoid infinite accelerations. This determines the relationship between N and the duration of the advancement, which is $\pi - 2\alpha$ by inspection of the figure. Consequently, the duration of the advancement increases with the number of stations, approaching a limit of 180° as the number of stations becomes very large. This has the advantage of making the advancement motion more gentle but the possible disadvantage of decreasing the duration of the period for which the output is stationary. The trade-off between these effects and with the desirability of avoiding gearing downstream of the indexing mechanism determine the choice of the number of stations. Gearing downstream of an indexing mechanism should be avoided due to the inaccuracy and uncertainty in position introduced by necessary backlash in the gear train. Gear backlash is not usually a problem if the gears are in uniform motion. However, the discontinuous motion output from an indexing mechanism and consequent reversals of acceleration result in slapping across the backlash interval. Hence, any speed reduction should be done upstream of the indexing mechanism.

The number of stations also determines the ratio of the center distance of the wheel axes to the pin radius and the outside diameter of the star wheel. By inspection of Fig. 5.31, the former ratio is $1/\sin\alpha$ and the latter is $1/\tan\alpha$.

Noting that $\alpha = \pi/N$, Eqs. (5.27)–(5.29), respectively, become for this more general case:

$$\phi = \tan^{-1}\left(\frac{\sin\alpha\sin\theta}{1 - \sin\alpha\cos\theta}\right) \tag{5.30}$$

$$\dot{\phi} = \dot{\theta}\sin\alpha\left(\frac{\cos\theta - \sin\alpha}{1 + \sin^2\alpha - 2\sin\alpha\cos\theta}\right) \tag{5.31}$$

$$\ddot{\phi} = -\dot{\theta}^2\frac{\sin\alpha\cos^2\alpha\sin\theta}{(1 + \sin^2\alpha - 2\sin\alpha\cos\theta)^2} \tag{5.32}$$

Spherical Geneva mechanisms allow indexed motion transfer between angulated shafts. More important, a large number of stations can be accommodated without losing positive locking action between advances.

Example

An indexing drive is to be driven by a synchronous electric motor turning at 360 rpm (the speed of a synchronous motor is locked to the alternating-current cycle frequency and so is essentially constant). The single pin driver is to turn a six-station Geneva wheel. Compute the following:

(a) The number of advances per second
(b) The angle through which the Geneva wheel advances during every revolution of the driving wheel
(c) The duration in seconds of the dwell in the output motion
(d) The peak angular velocity of the output shaft
(e) The peak angular acceleration of the output shaft

Solution

(a) The number of advances per second is the number of revolutions of the driver per second, which is $360/60 = 6$.
(b) The angle advanced is $2\alpha = 360°/N = 60°$, with N, the number of stations, being 6 in this case. Hence $\alpha = 30°$.
(c) The fraction of the cycle during which the output is locked (dwelling) is

$$\lambda = \frac{180 - 2\alpha}{360}$$

with α in degrees giving $\lambda = 1/3$. The duration of the complete cycle is $T = 1/6$ s from part (a). Hence the duration of the dwell is

$$\tau = \lambda T = 1/18 = 0.0555 \text{ s}$$

(d) Referring to Eq. (5.32), $\dot{\phi}$ is at its maximum value when $\theta = 0$. Also, for $N = 6$,

$$\sin\alpha = 0.5$$

so, substituting this value and $\theta = 0$ in Eq. (5.31),

$$\dot{\phi}_{\max} = \dot{\theta}$$

$\dot{\theta}$ is the angular velocity of the drive wheel, so

$$\dot{\theta} = 2\pi \times 6 = 37.70 \text{ rad/s}$$

Therefore,

$$\dot{\phi}_{\max} = 37.70 \text{ rad/s}$$

Note that ϕ is positive in the CCW direction while θ is positive in the CW direction (see Fig. 5.29). Therefore the positive values for both $\dot{\phi}$ and $\dot{\theta}$ indicate that the star wheel rotates in the opposite direction to the driver.
(e) It is necessary to determine the value of θ that maximizes $\ddot{\phi}$. A straightforward way to do this would be to plot Eq. (5.32) in the same way as in Fig. 5.30, but with $\alpha = 30°$. $\ddot{\phi}$ and the angle θ at which it occurs could then be read directly from the plot.
Alternatively, we can differentiate Eq. (5.32) to identify the extrema of $\ddot{\phi}$. Noting that $\dot{\theta}$ is constant,

$$\frac{d\ddot{\phi}}{dt} = \frac{-\dot{\theta}^3 \sin\alpha\cos^2\alpha}{(1 + \sin^2\alpha - 2\sin\alpha\cos\theta)^3}\{(1 + \sin^2\alpha - 2\sin\alpha\cos\theta)\cos\theta - 4\sin\alpha\sin^2\theta\}$$

and so

$$\frac{d\ddot{\phi}}{dt} = 0$$

when

$$(1 + \sin^2\alpha)\cos\theta - 2\sin\alpha\cos^2\theta - 4\sin\alpha\sin^2\theta = 0$$

Replacement of $\sin^2\theta$ by $1 - \cos^2\theta$ and rearrangement of the equation give

$$\cos^2\theta + \gamma \cos\theta - 2 = 0$$

where

$$\gamma = \frac{1 + \sin^2\alpha}{2 \sin\alpha} \tag{5.33}$$

The preceding equation can be treated as a quadratic equation in the variable $\cos\theta$. Solving for $\cos\theta$,

$$\cos\theta = \frac{-\gamma \pm \sqrt{\gamma^2 + 8}}{2}$$

It is possible to show that only the positive value of the square root gives a value of $\cos\theta$ with magnitude between 0 and 1 in the allowable range of α, $0 < \alpha < 60°$, so only that solution is valid. Hence, $\ddot{\phi}$ is at a maximum when

$$\theta = \pm \cos^{-1}\left(\frac{-\gamma + \sqrt{\gamma^2 + 8}}{2}\right) \tag{5.34}$$

where the $\pm$ sign now comes from inversion of the cosine, not from the quadratic solution. Equations (5.33) and (5.34) are of general validity for locating the maximal values of $\ddot{\phi}$. In the present case, substituting $\sin\alpha = 0.5$ in Eq. (5.33) gives

$$\gamma = 1.25$$

Hence Eq. (5.34) gives

$$\theta = \pm 22.90°$$

Substitution of these values into Eq. (5.32) gives

$$\frac{\ddot{\phi}}{\dot{\theta}^2} = \pm 1.372$$

Hence, since $\dot{\theta} = 6 \times 2\pi = 37.70$ rad/s, the peak angular acceleration is 1,950 rad/s^2.